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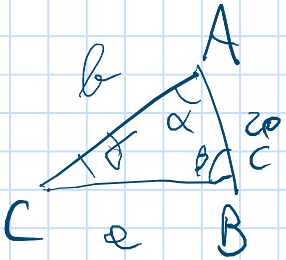

$c = AB = 20$

$ctg\,\hat{A} = \frac{3}{4} = ctg\,\alpha$

$\hat{C} = \frac{\pi}{6} = \gamma$

AC? CB?

Soluzione

t. dei seni $\frac{a}{sen\,\alpha} = \frac{b}{sen\,\beta} = \frac{c}{sen\,\gamma}$

$$\frac{c}{sen\,\gamma} = \frac{20}{sen\,30°} = \frac{20}{\frac{1}{2}} = 40$$

$$\frac{a}{sen\,\alpha} = \frac{c}{sen\,\gamma} \longrightarrow \frac{a}{sen\,\alpha} = 40$$

$$ctg\,\alpha = \frac{\cos\alpha}{sen\,\alpha} = \frac{3}{4} \Rightarrow tg\,\alpha = \frac{4}{3} \quad \alpha = arctg\left(\frac{4}{3}\right)$$

OPPURE:

$$\frac{\sqrt{1 - sen^2\alpha}}{sen\,\alpha} = \frac{3}{4} \qquad \frac{1 - sen^2\alpha}{sen^2\alpha} = \frac{9}{16}$$

$$\frac{1}{sen^2\alpha} - 1 = \frac{9}{16} \qquad \frac{1}{sen^2\alpha} = 1 + \frac{9}{16}$$

$$\frac{1}{\operatorname{sen}^2\alpha} = \frac{16+9}{16} \qquad \frac{1}{\operatorname{sen}^2\alpha} = \frac{25}{16}$$

$$\frac{1}{\operatorname{sen}\alpha} = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$$

ma $\operatorname{tg}\alpha > 0$
$\Rightarrow 0 < \alpha < \frac{\pi}{2}$

$$\Rightarrow \operatorname{sen}\alpha = \frac{4}{5}$$

$$c = 40 \cdot \operatorname{sen}\alpha = \overset{8}{\cancel{40}} \cdot \frac{4}{\cancel{5}} = 32$$

$$\beta = \pi - \alpha - \gamma \qquad \gamma = \frac{\pi}{6}$$

$$\operatorname{sen}\beta = \sin(\pi - (\alpha+\gamma)) = \sin(\alpha+\gamma) =$$

$$= \sin\alpha\cos\gamma + \operatorname{sen}\gamma\cos\alpha = \frac{4}{5}\,\frac{\sqrt{3}}{2} + \frac{1}{2}\cos\alpha$$

$$\cos\alpha = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\operatorname{sen}\beta = \frac{2}{5}\sqrt{3} + \frac{1}{2}\,\frac{3}{5} = \frac{4\sqrt{3}+3}{10}$$

$$b = \operatorname{sen}\beta \cdot \frac{c}{\operatorname{sen}\gamma} = \frac{4\sqrt{3}+3}{10}\,40 = 4(4\sqrt{3}+3)$$

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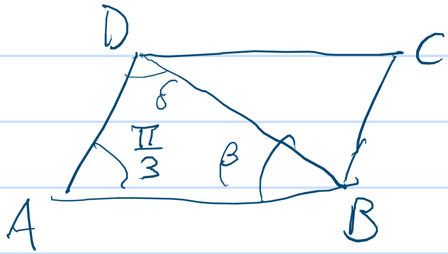

$BD = 12$

$\hat{A} = \widehat{DAB} = \frac{\pi}{3}$

Considero il triangolo $\overset{\triangle}{DAB}$

Per il t dei seni:

$$\frac{DB}{\operatorname{sen}\hat{A}} = \frac{DA}{\operatorname{sen}\beta} = \frac{AB}{\operatorname{sen}\delta} \qquad \beta = \widehat{ABD} = \frac{\pi}{4}$$

$$\Rightarrow \quad DA = \frac{\operatorname{sen}\beta}{\operatorname{sen}\hat{A}} \cdot DB = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} \cdot 12 =$$

$$= \frac{\sqrt{2}}{\sqrt{3}}\, 12 = \frac{\sqrt{2}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}}\, 12 = \frac{\sqrt{6}}{3} \cdot 12 = 4\sqrt{6}$$

$$\delta = \pi - \hat{A} - \beta$$

$$\operatorname{sen}\delta = \operatorname{sen}\left(\pi - (\hat{A} + \beta)\right) = \operatorname{sen}(\hat{A} + \beta) =$$

$$= \operatorname{sen}\hat{A}\cos\beta + \operatorname{sen}\beta\cos\hat{A} =$$

$$\operatorname{sen}\delta = \operatorname{sen}\left(\pi - (\hat{A} + \beta)\right) = \operatorname{sen}(\hat{A} + \beta) =$$

$$= \operatorname{sen}\hat{A}\cos\beta + \operatorname{sen}\beta\cos\hat{A} =$$

$$\hat{A} = \frac{\pi}{3} \qquad \beta = \frac{\pi}{4}$$

$$\operatorname{sen}\delta = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$AB = \operatorname{sen}\delta \cdot \frac{DB}{\operatorname{sen}\hat{A}} = \frac{\sqrt{6}+\sqrt{2}}{\cancel{4}_{\,1}} \quad \frac{\cancel{12}^{\,3}}{\frac{\sqrt{3}}{2}} =$$

$$= 2\left(\sqrt{6}+\sqrt{2}\right)\sqrt{3} = 2\sqrt{6}\sqrt{3} + 2\sqrt{2}\sqrt{3} =$$

$$= 2\sqrt{2}\sqrt{3}\sqrt{3} + 2\sqrt{2}\sqrt{3} = 6\sqrt{2} + 2\sqrt{6}$$

$$AB + AD = 6\sqrt{2} + 2\sqrt{6} + 4\sqrt{6} = 6\sqrt{2} + 6\sqrt{6} =$$

$$= 6\sqrt{2}\left(1+\sqrt{3}\right)$$

$$p = 2(AB + AD) = 12\sqrt{2}\left(1+\sqrt{3}\right)$$

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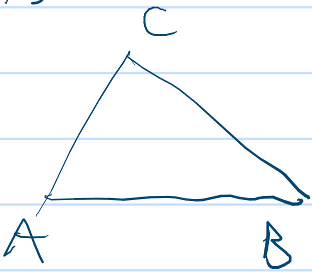

$AB = 10\sqrt{7}$

$\sin\hat{A} = \frac{3}{5}$

$\cos\hat{C} = -\frac{3}{4}$ $\quad \hat{C} > \frac{\pi}{2}$

$$\Rightarrow \sin\hat{C} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$\frac{CB}{\sin\hat{A}} = \frac{AB}{\sin C} \rightarrow CB = \frac{\sin\hat{A}}{\sin\hat{C}} AB =$$

$$= \frac{3}{\cancel{5}_{1}} \frac{4}{\cancel{\sqrt{7}}} \cancel{10}^{2}\cancel{\sqrt{7}} = 24$$

$$\sin\hat{B} = \sin(\pi - \hat{A} - \hat{C}) = \sin(\hat{A} + \hat{C}) =$$

$$= \sin\hat{A}\cos\hat{C} + \sin\hat{C}\cos\hat{A} =$$

$$\frac{3}{5}\left(-\frac{3}{4}\right) + \frac{\sqrt{7}}{4}\cos\hat{A}$$

$$\cos\hat{A} = +\sqrt{1 - \sin^2\hat{A}} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5} \qquad \hat{A} < \frac{\pi}{2}$$

$$\sin\beta = -\frac{9}{20} + \frac{\sqrt{7}}{4}\frac{4}{5} = -\frac{9}{20} + \frac{\sqrt{7}}{5}$$

$$AC = \operatorname{sen}\beta \cdot \frac{AB}{\operatorname{sen}\hat{C}} = \left(\frac{\sqrt{7}}{5} - \frac{9}{20}\right) \frac{10\sqrt{7}}{\frac{\sqrt{7}}{4}} =$$

$$= 40\left(\frac{\sqrt{7}}{5} - \frac{9}{20}\right) = 8\sqrt{7} - 18 = 2(4\sqrt{7} - 9)$$